Game Theory:
What Is This?

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Despite its frivolous name, Game Theory is a fairly serious branch of applied mathematics that concerns itself with the study of multi-person interdependent decision-making. It is a set of techniques for analyzing strategic situations; that is, situations in which at least two agents make decisions that affect one another's welfare. The insights offered by the theory can be therefore applied across a wide range of disciplines including economics, political science, anthropology, sociology, and biology.

A *game* refers to a strategic situation that involves at least two rational intelligent individuals called *players*. A *rational* player is one who consistently makes decisions in pursuit of some well-defined objectives. An *intelligent* player is one who knows everything we know about the game and who can make the same inferences as we do.

We shall spend the next few classes developing the idea of rationality from primitives, but a few words are in order here. The fundamental result of decision theory, which forms the core of game theory as well, is that each player's objective is to maximize the expected value of his own payoff. These payoffs are measured on some *utility* scale, which is simply a numeric representation of each outcome that can be realized through the actions of the player. In other words, we assume that given several *outcomes*, individuals have *preferences* over them that allow them to rank the outcomes with respect to each other. That is, for each pair of outcomes, a player can say whether he likes one better than the other or whether he is indifferent between the two. The most remarkable result due to von Neumann and Morgenstern is that given very weak assumptions about these preferences, there exists a way of assigning utility numbers to the various outcomes such that the decision maker would always choose the option that maximizes the expected utility. We shall develop this *expected utility maximization theorem* next time in some detail. It is the cornerstone of game theory.

Very often, individuals may be uncertain about the outcomes. Expected utilities can then be computed only if these uncertain events have probabilities assigned their occurrence. Sometimes it is not possible to assign objective probabilities to some events, but important results in decision-theory show that in these cases rational individuals would be able to assess subjective probabilities necessary to compute expected payoffs.

Unlike decision-theory, in which a single individual maximizes his payoff given his subjective probabilities assigned to various outcomes, game theory deals with multi-person decision-making. The difficulty of this subject matter should be now obvious. If player 1's payoff depends on what player 2 does and vice versa, then player 1 must take into account what player 2 is going to do. Given that player 2 is also rational, player 1 would attempt to guess what she would do, which immediately leads to the realization that player 2 would try to guess what player 1 is going to do. The optimal decision of each player depends on the optimal decision of the other player. When rational individuals interact, their behavior must be analyzed simultaneously.

More recently, game theorists have become increasingly interested in dynamic models that go beyond the strict rationality assumptions of classical game theory. Behavioral game theory is a blend of innovative techniques that model adaptive behavior by players who are not strictly rational, who have limited memories, cognitive capacities, and foresight. The theory of learning in games is still in its infancy and the dynamic models that incorporate it are quite demanding. Still, many of their results are corroborated by sophisticated experimental evidence. Most tellingly, these models demonstrate that frequently these limited-rationality agents learn to play the games in optimal ways. That is, the stable states of these dynamic systems often (although not always) converge to equilibria derived from classical game theory models. Thus, despite its seemingly excessive demand of perfect rationality, classical game theory is an extremely useful tool for the analysis of conflict.

The future of game theory is in these low-rationality evolutionary models but the basics that we learn here will remain quite useful for a long time. I should note that strict rationality is not as excessive an assumption as many may think. In particular, one should be extremely wary of any theory that assumes that individuals systematically make mistakes or fail to understand the situation for a long time. Learning by individuals will invariably invalidate such theories.